

As  $z \rightarrow \infty$ , Eq. (15) tends asymptotically to zero, and for values of the argument  $\leq -3$  it is close to unity.

An approximate solution of the initial problem in Eqs. (4) and (15) can be constructed as follows. For fixed  $t$ , the concentration  $\Theta$  is equal to unity on a section from 0 to  $x_*$ , where  $x_*$  is the value of  $x$  for which  $z/2\sqrt{Dt} = -3$ . For  $x > x_*$ , the distribution of the concentration is described by Eq. (15).

#### NOTATION

$\Theta$ , concentration of material in porous medium;  $a$ , concentration of material in sorption layer;  $\Gamma$ , Henry constant;  $D$ , diffusion coefficient;  $m_0$ , porosity;  $\tau$ , delay time;  $t, x$ , time and space coordinates;  $u$ , filtration velocity.

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#### CONVECTIVE MASS EXCHANGE OF A CONE AND DYNAMIC CHARACTERISTICS OF AN ELECTRODIFFUSION ANEMOMETER IN A NONLINEARLY VISCOUS FLUID FLOW

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The results of a theoretical and experimental investigation of the nonstationary mass exchange of a cone in a nonlinearly viscous fluid flow are presented. The amplitude - frequency characteristics of an electrodiffusion anemometer are plotted from the results obtained.

Electrodiffusion anemometry, an electrochemical method now widely used for studying heat- and mass-exchange processes, forms the basis of a new trend in the measurement of the velocities of fast-flowing media [1-5].

The essence of this method of measurement is that the mass transfer in the electrolyte solution is due to the diffusion of electrically charged particles, i.e. to the simultaneous transfer of mass and electrical charge. It is then possible, depending on the magnitude of the limiting diffusion current, to make a judgment about the intensity of mass transfer towards the transducer and, ultimately, to determine the local instantaneous velocity of the fluid flow. The transducer-electrode used in these measurements usually has a conical, wedge-shaped, cylindrical, or spherical surface, and a number of problems related to the selection of optimum electrode dimensions - its sensitivity, the nature of the dependence of the current on velocity, and the statistical and dynamic characteristics - are determined by the principles governing stationary and nonstationary convective diffusion. None of these problems has been studied in any depth so far even with reference to the simplest case - the flow of purely viscous linear fluid electrolytes around an electrode.

In practice, however, it is often a question of fluids possessing non-Newtonian properties and this cannot be taken into account in calculating the convective diffusion processes. Thus, a theoretical and experimental

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investigation of the mass exchange of a rotating disk electrode in an electrolyte with an exponential rheological equation of state shows significant special characteristics in the convective diffusion processes in these fluids [6].

The subject of this paper is a theoretical and experimental investigation of convective mass exchange and of the dynamic characteristics of a conical electrode around which there is generally a flow of an incompressible nonlinear purely viscous fluid with an exponential rheological equation of state [7]:

$$\tau_{ij} = -p\delta_{ij} + k \left| \frac{1}{2} \dot{e}_{rm} \dot{e}_{mr} \right|^{\frac{n-1}{2}} \dot{e}_{ij}. \quad (1)$$

The flow velocity range is considered as stationary, and a sudden change in concentration is generated at the solid — fluid interface. The parameters of the rheological equation of state (1) and the physical properties of the fluid are assumed to be independent of the concentration.

The mass exchange is treated as low-intensity in the sense that the concentration range of the diffusing additive does not influence the dynamic characteristics of the flow. It is also assumed that the thickness of the boundary layer is much lower than the radius of curvature of the cone. The x axis is aligned along the generating line of the cone and the y axis, perpendicular to it.

The mathematical formulation of the problem is represented by a system of equations [8]:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = V \frac{dV}{dx} + \frac{k}{\rho} \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right)^n, \quad (2)$$

$$\frac{\partial(ux)}{\partial x} + \frac{\partial(vx)}{\partial y} = 0, \quad (3)$$

$$\frac{\partial C_1}{\partial t} + u \frac{\partial C_1}{\partial x} + v \frac{\partial C_1}{\partial y} = D \frac{\partial^2 C_1}{\partial y^2} \quad (4)$$

with initial and boundary conditions

$$u(x, 0) = v(x, 0) = 0; u(x, \infty) = V = bx^m; \quad (5)$$

$$C_1(x, y, 0) = C_0; C_1(x, 0, t) = 0; C_1(x, \infty, t) = C_0. \quad (6)$$

The parameter m in the law governing the velocity of the potential flow is related to the magnitude of the half-angle  $\alpha$  at the vertex of the cone by an approximate relationship [9]:

$$m = \frac{4}{\pi^2} \alpha^2. \quad (7)$$

The dynamic problem (2), (3), and (5) is self-similar. The velocity distribution being sought in the boundary layer is found from the formulas

$$u = bx^m F'(\eta), \quad (8)$$

$$\eta = \left( \frac{b^{2-n} \rho}{k} \frac{m(2n-1) + n + 2}{n+1} \right)^{\frac{1}{1+n}} yx^{\frac{m(2-n)-1}{n+1}} \equiv yx^{\frac{m(2-n)-1}{n+1}} M^{\frac{1}{1+n}},$$

$$v = -bM^{-\frac{1}{1+n}} x^{\frac{m(2n-1)-n}{n+1}} \left[ \frac{m(2n-1) + n + 2}{n+1} F + \frac{m(2-n)-1}{n+1} \eta F' \right]. \quad (9)$$

The equation of convective diffusion is solved by using the fact that in the case under examination the diffusion Pr numbers are extremely large, so that it is admissible to limit oneself to the linear part of the velocity profile:

$$\frac{u}{V} = \beta_1 y = a\eta, \quad \beta_1 = \frac{\partial(u/V)}{\partial y} \Big|_{y=0}, \quad (10)$$

$$a = \frac{\partial(u/V)}{\partial \eta} \Big|_{\eta=0} = F''(0).$$

The dynamic form parameter in Eq. (10) is found by using a Stepanov — Mangler transform for an "exponential" fluid [6] and by using the results of exact and approximate solutions of the equations for the boundary layer on a flat surface [8, 10]. Values of  $F''(0) = a$  for a number of m and n values are given in Table 1.

TABLE 1. Values of  $F''(0) = a$  and  $\lambda^{1/2}(0)$  for Different  $m$  and  $n$

$n$	$m$						
	0,1	0,2	0,4	0,5	0,6	0,8	1,0
$F''(0) = a$							
0,2	0,0922	0,1669	0,3086	0,3798	0,4525	0,6059	0,7746
0,4	0,1532	0,252	0,4166	0,4908	0,5619	0,6977	0,8285
0,6	0,219	0,336	0,5136	0,5864	0,6538	0,7751	0,883
0,8	0,2853	0,4145	0,5956	0,6668	0,73	0,8389	0,9308
1,0	0,3486	0,4857	0,6663	0,7342	0,793	0,8913	0,9726
$\lambda^{1/2}(0)$							
0,2	2,7054	2,648	2,5359	2,4801	2,4236	2,3043	2,1635
0,4	2,6861	2,6372	2,5412	2,4935	2,4453	2,3454	2,2349
0,6	2,6718	2,6292	2,5452	2,5034	2,4614	2,3752	2,2829
0,8	2,6617	2,6229	2,5483	2,5112	2,4739	2,398	2,3182
1,0	2,6518	2,6179	2,551	2,5174	2,4839	2,4159	2,3454

By changing from  $x, y,$  and  $t$  coordinates to  $x, \psi,$  and  $t$  coordinates in (4) and by then introducing the variables

$$C = \frac{C_0 - C_1}{C_0}, \tau = \frac{B^{4/3}t}{xND} = \frac{Vt}{x} \text{Pr}_x^{-1/3} A^{2/3},$$

$$\omega = \frac{\Psi^{1/2}}{x^E \left(\frac{3B}{4E}\right)^{1/3}}, \text{ where } \text{Pr}_x = \frac{Vx}{D} \left(\frac{V^{2-n} \rho x^n}{k}\right)^{\frac{2}{1-n}} \quad (11)$$

is a generalized diffusion Prandtl number,

$$\Psi = bM^{-\frac{1}{1+n}} x^{\frac{m(2n-1)+n+2}{n+1}} F(\eta) \quad \text{is a point function,}$$

$$A = \frac{a(3m+5n+4)}{18(n+1)} \left[ \frac{m(2n-1)+n+2}{n+1} \right]^{\frac{1}{1-n}},$$

$$E = \frac{3m+5n+4}{6(n+1)}, \quad H = \frac{2n-6m+4}{3(n+1)}, \quad B = [2abM^{\frac{1}{1-n}}]^{1/2} D,$$

we obtain

$$\frac{\partial C}{\partial \tau} - \frac{3H}{E} \omega \tau \frac{\partial C}{\partial \tau} - 3\omega^2 \frac{\partial C}{\partial \omega} = \frac{\partial^2 C}{\partial \omega^2}, \quad (12)$$

$$C(\omega, 0) = 0, \quad C(0, \tau) = 1, \quad C(\infty, \tau) = 0. \quad (13)$$

A method of solving (12) and (13) is set out in detail in [11]. A solution of the Laplace-transformed equation (12) is sought in the form of a series containing a certain initially known function  $\lambda(\omega)$  which is then sought from the solution to the stationary problem. By using the results of [11], we obtain

$$-C'(0, \tau) = \left\{ \frac{\exp[-\lambda(0)\tau]}{(\pi\tau)^{1/2}} + \lambda^{1/2}(0) \operatorname{erf}[\lambda(0)\tau]^{1/2} - \sum_{i=0}^{\infty} \frac{g'_i(0)}{\lambda^{1/2}(0)} \cdot \frac{\Gamma[\lambda(0)\tau, l/2]}{\Gamma(l/2)} \right\}, \quad (14)$$

$$j_x = D \frac{\partial C}{\partial y} \Big|_{y=0} = \left(\frac{1}{6} EB^2\right)^{1/3} C_0 x^{\frac{2n-6m+4}{6(n+1)}} C'(0, \tau), \quad (15)$$

where  $g'_i(0)$  are given by formulas (33) in [11] and  $\lambda(0)$  is found from the equation [11]

$$-C'(0, \tau) = -C'_{st}(0) = \lambda^{1/2}(0) - \sum_{i=0}^{\infty} \frac{g'_i(0)}{\lambda^{1/2}(0)}. \quad (16)$$

Values of  $\lambda(0)$  for different  $m$  and  $n$  are given in Table 1.

It must be noted that the series in the right-hand part of (14) and (16) is "semidivergent" and its sum is found with the help of the Euler transform [12] which improves the divergence. An analysis of (15) shows that the characteristic stationary-mode establishment time of the process is determined by the relationship

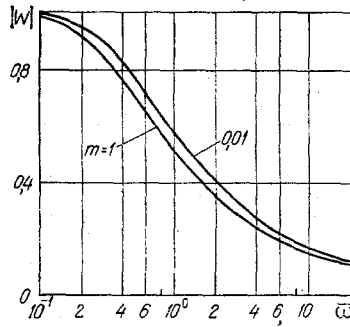


Fig. 1. Dependence of the frequency characteristic modulus on the dimensionless frequency for a conical transducer.

$$T_x = \frac{L \text{Pr}_L^{1/3}}{V\lambda(0)A^{2/3}} \bar{x}^{\frac{2n+4-6m}{3(n+1)}}, \quad \bar{x} = \frac{x}{L}. \quad (17)$$

With the characteristic time it is possible to assess the influence of the parameters of the fluid, the dimensions of the body, and the velocity of flow on the progress of the transient process. Thus, calculations show, other conditions being equal, that  $T_x$  increases as the pseudoplastic properties of the medium are strengthened and as  $m$  is reduced, with the influence of  $m$  becoming greater as  $n$  increases.

An expression for the total mass flow is obtained by integrating (15) over the cone surface:

$$I = 2\pi \sin \alpha \int_0^L j_x x dx = 2\pi \sin \alpha \left[ \frac{1}{6} EB^2 \right]^{1/3} C_0 \int_0^L x^{-\frac{2n-6m+4}{6(n+1)}} C'(0, \tau) dx, \quad (18)$$

$$I_{st} = \frac{C_0 2\pi \sin \alpha 3(n+1)}{1/3\Gamma(1/3)(3m+5n+4)} L^{\frac{3m+5n+4}{3(n+1)}} v_0^{\frac{1}{1-n}} D^{2/3} \left\{ 1/3 Ea \left[ \rho/k \frac{m(2n-1) + n + 2}{n+1} \right]^{1/3} \right\}^{1/3}, \quad (19)$$

where  $L$  is the generating line of the cone.

When measurements are made in a non-Newtonian fluid characterized, as a rule, by a comparatively high viscosity, by high-value diffusion  $\text{Pr}$  numbers, and by micro- and macroinhomogeneities, the preferred shapes of measuring electrodes are the wedge or the cone. The use of wire cylindrical electrodes, which can have extremely small dimensions, and, thus, extremely small inertia ratings, is not very widespread because the measuring surface has a low mechanical strength and is frequently clogged with dirt.

The frequency characteristics of a flat viscous friction shearing-stress transducer are determined in papers [13-15]. The simplifications of the diffusion equation used in these papers, which are related to the small dimensions, the shape of the electrode, and its location, are confirmed for the case in which the variation in the thickness of the hydrodynamic boundary layer running the length of the electrode can be ignored. This occurs when measurements are made by a microelectrode fairly far removed from the initial point of growth of the hydrodynamic boundary layer. When the velocity is measured by, for example, a conical electrode, the initial point of growth of the dynamic and diffusion layers coincide and the above-mentioned assumptions are, in this case, hardly justified.

The transfer function of the linear system under investigation can be found using a transient function — the reaction of the system to a step-by-step change in the input signal [16], in our case, to a step-by-step change in the velocity field. The solution to such a problem is related to well-known difficulties which are aggravated further by the need to take into account the nonlinear properties of pseudoplastic and dilatant fluids. The problem can, however, be simplified considerably if it is taken into account that with high  $\text{Pr}$  numbers the stabilization time of the hydrodynamic boundary layer is small enough to be ignored compared with the stabilization time of the diffusion boundary layer and that the nonstationary convective diffusion governs the whole course of the transient process [21, 22]. In this case, the step-by-step change in the velocity can be identified with a step-by-step accretion of "fresh" portions of the substance and the electrode current generated by the step-by-step change in the gradient of the surface concentration can be taken as the transient characteristic.

The results obtained above [(18) and (19)] are used to determine the transfer function:

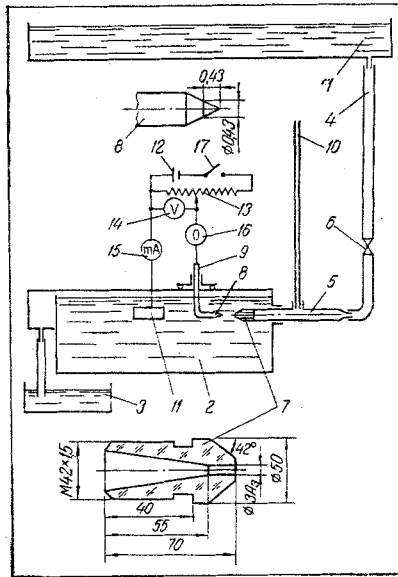


Fig. 2. Experimental installation diagram: 1) supply tank; 2) container; 3) overflow tank; 4) hose pipe; 5) supply pipe; 6) valve; 7) nozzle; 8) working medium; 9) coordinating device; 10) piezometer; 11) anode; 12) galvanic cell; 13) rheostat; 14) voltmeter; 15) micrometer; 16) loading resistance; 17) switch.

$$W(s) = \frac{2\pi \sin \alpha \left[ \frac{1}{6} EB^2 \right]^{1/3} [1/3\Gamma(1/3)]^{-1} \int_0^L x^{-\frac{2n-6m+4}{6(n+1)}+1} dx}{2\pi \sin \alpha \left[ \frac{1}{6} EB^2 \right]^{1/3} \int_0^L x^{-\frac{2n-6m+4}{6(n+1)}+1} \left\{ \lambda^{1/2}(0)(sT_x+1)^{1/2} - \sum_{l=0}^{\infty} \frac{g_l(0)}{\lambda^{l/2}(0)} (sT_x+1)^{-l/2} \right\} dx} \quad (20)$$

By substituting  $i\omega$  for  $s$  in (20) we obtain an expression for the frequency characteristic, which, after slight transformation, takes the form

$$W(i\bar{\omega}) = \left\{ [1/3\Gamma(1/3)]^{-1} \int_0^1 \bar{x}^{-\frac{2n-6m+4}{6(n+1)}+1} d\bar{x} \right\} \left\{ \int_0^1 \bar{x}^{-\frac{2n-6m+4}{6(n+1)}+1} \times \right. \\ \left. \times \left[ \lambda^{1/2}(0)(i\bar{\omega} \bar{x}^{-\frac{6m-2n-4}{3(n+1)}+1})^{1/2} - \sum_{l=0}^{\infty} \frac{g_l(0)}{\lambda^{l/2}(0)} (i\bar{\omega} \bar{x}^{-\frac{6m-2n-4}{3(n+1)}+1})^{-l/2} \right] d\bar{x} \right\}^{-1}, \quad (21)$$

where

$$\bar{x} = \frac{x}{L}, \quad \bar{\omega} = 2\pi f T_L = \omega T_L, \quad T_L = \frac{L P r_L^{1/3}}{V \lambda(0) A^{2/3}}. \quad (22)$$

The real and imaginary parts of  $W(i\bar{\omega})$  are calculated by formula (21) on a Minsk-22 computer. The results of calculating the frequency characteristic modulus  $|W|$  are shown in Fig. 1, showing that for a fixed value of  $m$  the change in the index  $n$  has virtually no influence on the behavior of  $|W|$  as a function of

TABLE 2. Rheological Parameters of Solutions under Investigation

0.025 M $K_4[Fe(CN)_6]$ / [ $K_3[Fe(CN)_6]$ + 0.5 M $K_2SO_4$ + Na-CMC % solution	$\rho \cdot 10^{-3}$ kg/m <sup>3</sup>	$k \cdot 10^3$ kg/sec <sup>n-2</sup> / m	$n$	$D \cdot 10^9$ m <sup>2</sup> / sec
0	1,015	1,16	1	0,625
1	1,032	2,15	0,94	0,612
1,5	1,04	870	0,604	0,469

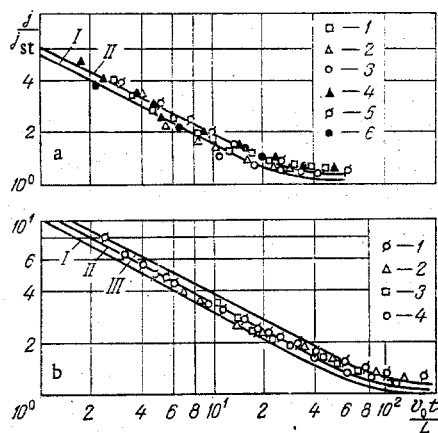


Fig. 3

Fig. 3. Nonstationary convective mass exchange of a conical electrode ( $L = 0.537 \cdot 10^{-3}$  m;  $2\alpha = 53^\circ$ ): a) I, 0% Na-CMC [1]  $v_0 = 0.42$ ; 2) 0.82; 3) 3.9 m/sec; II, 1% Na-CMC [4]  $v_0 = 0.35$ ; 5) 0.7; b) 1.97 m/sec; 1.5% Na-CMC; I,  $v_0 = 1.8$  m/sec; II, 1.8; III, 0.98; 1,  $v_0 = 0.4$ ; 2, 0.98; 3, 1.8; 4, 2.8 m/sec.

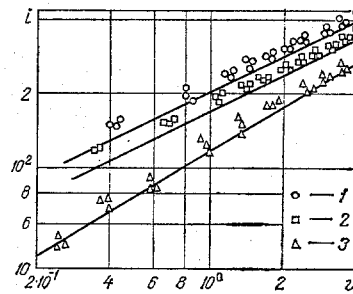


Fig. 4

Fig. 4. Stationary convective mass exchange of a conical electrode: ( $L = 0.53 \cdot 10^{-3}$  m;  $2\alpha = 53^\circ$ ): 1) 0% Na-CMC; 2) 1% Na-CMC; 3) 1.5% Na-CMC;  $i$ ,  $\mu\text{A}$ ,  $v$ , m/sec.

the dimensionless frequency  $\bar{\omega}$ . According to [19], the relationship between the spectral densities of the current and velocity fluctuations can be represented, using the frequency characteristic modulus, by the formula

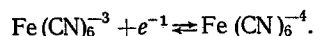
$$S_{u_i} = S_u |W|^2, \quad (23)$$

where  $S_{u_i}$  is the spectral density of the velocity fluctuations, recalculated for the spectral density of the diffusion current without making allowance for the inertia of the transducer.

From (22) and Fig. 1, it is clear that the sensitivity of the electrodiffusion transducer falls as the frequency of the perceptible signal increases, but the parameter governing the frequency bandwidth of the transducer is the time constant  $T_L$ , and as this is reduced, the bandwidth is increased. The magnitude of  $T_L$  is dependent on  $m$ , which is related directly (7) to the angle at the vertex of the conical transducer-electrode, the length of the generating line, and the properties of the fluid, and, as can be seen from (17) and (22),  $T_L$  is analogous to the characteristic stationary-mode establishment time of the process which has been examined already when  $\bar{x} = 1$ .

The mass exchange of the cone is investigated experimentally using the electrodiffusion method, which makes it possible to effect a practically intermittent change in the gradient of the concentration at the solid - solution interface. The electrode polarization achieved during the experiment for all values of  $t$  ensures that the boundary conditions are satisfied fairly precisely in accordance with (6). The high degree of reliability of the electrodiffusion method of investigating fast-flowing transient processes is shown in [1, 6].

Three solutions are used as working fluids: a pure electrolyte, a 1% solution of a Na-CMC (carboxymethylcellulose) polymer (first batch), and a 1.5% solution of Na-CMC (second batch) in the presence of an electrolyte. A 0.025 M aqueous solution of ferrocyanide  $K_4[Fe(CN)_6]/K_3[Fe(CN)_6]$  is used as the electrolyte, and an oxidation-reduction reaction takes place in it:



A "background electrolyte," 0.5 M  $K_2SO_4$ , is added to the electrolyte to reduce ion migration under the action of the electric field. The basic merits of this system are the high electrochemical reaction rate, which makes possible its use in studying transient processes, and the absence of reaction-product liberation.

It is a well-known fact that aqueous solutions of Na-CMC possess pseudoplastic properties which are

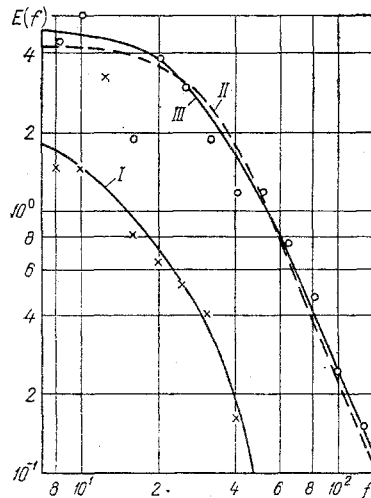


Fig. 5. Energy spectrum of the longitudinal component of the fluctuation velocity on the axis of a turbulent submerged stream ( $x/D = 30$ ,  $v_0 = 0.45$  m/sec;  $n = 1$ ): I) data obtained using a conical electrode; II) data from I re-calculated taking into account (21) and (23); III) data obtained using a wedge-shaped electrode.  $f$ , Hz.

maintained in the presence of the above-mentioned electrolyte [17, 18]. The rheological properties of the solutions are investigated on a "Rheotest" rotary viscometer. Table 2 shows that the 1.5% solution of Na-CMC possesses marked pseudoplastic properties, while the 1% solution possesses extremely weak pseudoplastic properties. The coefficients of  $\text{Fe}(\text{CN})_6^{3-}$  ion diffusion are measured by the "surge current" method [20].

An electrode with a conical working surface is tested in a submerged stream with a discharge rate which can be regulated in the 0.3-4 m/sec range. Figure 2 gives an installation diagram.

The electrical circuit is also given in this diagram. The cathode is a conical electrode 8 and the anode is a platinum plate 11 which has an area  $10^3$  times greater than the area of the cathode. The process is recorded by an NI02-type loop oscillograph 16 and the time markings are recorded at the same time at a frequency of 1000 Hz. The conical electrode 8 is fixed in the potential center of the submerged flow at 3-4 cm from the edge of the nozzle 7 in the flat velocity-profile region. In this cross section the intensity of the turbulence of the incoming stream is less than 0.5%. The polarization voltage is fed to the electrodes intermittently by closing the switch 17, thus generating a transient convective diffusion process.

The experimental data are processed in the form  $(i/i_{st}) = (I/I_{st}) = f(v_0 t/L)$  for nonstationary diffusion current measurements and in the form  $i = f(v_0) = I_{st} z F$  for stationary measurements. Figure 3a shows the results of measuring nonstationary diffusion flows for different velocities of flow of the pure electrolyte and the 1% solution of Na-CMC polymer in the presence of the electrolyte which is a virtually Newtonian fluid, as is shown by viscometry. Calculations based on the formulas (18) and (19) are shown by solid lines. It is clear that the experimental data are in good agreement with the theoretical relationships. The data given in Fig. 3b represent the results of an experiment on the nonstationary mass exchange of a cone in a 1.5% solution of Na-CMC possessing pseudoplastic properties. These results are also in good agreement with the calculations. The layering of the curves in Fig. 3b for different values of  $v_0$  is due to the fact that the generalized Pr number (II) is dependent on the velocity when  $n \neq 1$ . It is also clear from Fig. 3a, b that the stationary-mode establishment time of the nonstationary process is greater in the 1.5% Na-CMC solution than in the pure electrolyte. Calculations show that the time constant of the transducer operating in this solution is increased by a factor of 2-4 as a function of the velocity of the incoming flow compared with  $T_L$  in the pure electrolyte, i.e., the dynamic characteristics of the transducer in the electrolyte with polymer additives are impaired somewhat.

Figure 4 represents the results of the measurement of a stationary diffusion flow in the conical electrode circuit for different fluids and different velocities of flow around it. The solid line gives the figures calculated from formula (19). It is clear that a change in the parameter of non-Newtonian behavior of the fluid gives rise to a change in the magnitude of the current and in the nature of the  $i(v_0)$  relationship. Formula (19) shows that the tangent of the slope angle of the straight line  $i(v_0)$  in logarithmic coordinates is equal to  $1/(n+1)$ , i.e. it is greater for pseudoplastic fluids ( $n < 1$ ) than for the pure electrolyte ( $n = 1$ ). This is corroborated satisfactorily by the experiment.

In addition to measuring the nonstationary diffusion current and finding the stabilization time of the stationary state, the dynamic properties of the conical transducer-electrode are evaluated by comparing the spectral densities of fluctuations in the longitudinal component of the flow velocity on the axis of the turbulent submerged flow. Data obtained using the conical electrode under consideration ( $L \approx 0.74 \cdot 10^{-3}$  m  $\sim 0.2$ ) and recalculated taking into account its amplitude-frequency characteristic (21) and (23) are compared with data obtained using a wedge-shaped electrode which is virtually inertia-free in relation to extremely small reaction-area dimensions within the range of frequencies under investigation. Analysis shows that spectra plotted from measurements taken with the conical transducer and taking into account (21) and (23) are in good agreement with spectra measured using the wedge-shaped transducer. Figure 5 shows the results of a comparison of data from measurements in the  $x/D = 30$  cross section (data obtained using the conical electrode are not given in the 50-200-Hz range in the figure).

It should be noted that the feasibility of using the nonstationary diffusion current as the transient characteristic is also tested by calculating the dynamic characteristics of an electrodiffusion friction transducer. A comparison for  $m = 0$ ,  $n = 1$  with the amplitude-frequency characteristic obtained by a more accurate method by the authors of [14, 15] shows that the maximum divergence is less than 3 dB. This accuracy is quite acceptable for the practical use of the method proposed for calculating the amplitude-frequency characteristic.

#### NOTATION

$C_1$ , concentration;  $C_0$ , concentration in flow;  $x, y$ , coordinates oriented along the generating line of the cone and perpendicular to it, respectively;  $t$ , time;  $\Psi$ , current function;  $u, v$ , velocity components along  $x$  and  $y$  axes, respectively;  $V$ , velocity at boundary-layer limit;  $v_0$ , incoming flow velocity;  $D$ , diffusion coefficient;  $n$ , index of non-Newtonian behavior of fluid;  $m$ , parameter in law governing velocity at boundary-layer limit;  $k$ , degree of consistency;  $\eta$ , self-similar constant;  $\tau, \omega, C$ , dimensionless constants: time, coordinate, and concentration, respectively;  $s$ , Laplace-transform parameter;  $\tau_{ij}$ , stress-tensor component;  $\dot{\epsilon}_{ij}$ , strain-rate-tensor component;  $\delta_{ij}$ , Kronecker symbol;  $W$ , transfer function;  $f$ , frequency;  $z$ , electrovalence number;  $F$ , Faraday number.

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## MECHANICAL BEHAVIOR OF UNSTABLE EMULSIONS

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Results of experimental studies of the rheologic properties of unstable macroemulsions of the water — oil type are presented. A region of non-Newtonian behavior of the test emulsions is identified.

The rheologic properties of emulsions are far from completely studied, since emulsions are systems difficult to investigate.\* This is associated with the fact that the dispersed phase is a deformable fluid. The degree of deformation depends on the globule size and on the shear rate. Special difficulties arise in studies of the rheologic properties of unstable emulsions.

This paper is devoted to a presentation of experimental methods and results for a study of the rheology of unstable macroemulsions of the water — oil type. The emulsions were produced by turbulent mixing of transformer oil and an aqueous solution of 4411 disolvane at a concentration of 0.02 wt. % during their joint flow in a horizontal cylindrical pipe.

Differing degrees of emulsion dispersion were achieved by varying the mean flow velocity ( $w = 1.3$  m/sec and  $w = 1.7$  m/sec). The content of the dispersed phase varied from 0 to 0.35; stratification of the flow occurred at large  $\Phi$  with subsequent inversion of emulsion phases.

Since the emulsions studied were unstable, it was not possible to investigate their viscous properties under static conditions. Viscosity measurements of such emulsions were accomplished directly in the flow by means of a capillary method. The essentials of the method developed for this purpose are illustrated by the scheme shown in Fig. 1.

The glass capillary 2 was inserted in the experimental pipe 1 with an internal diameter of 39.4 mm along which the unstable emulsion flowed. The capillary tube was held in position by means of a brass fitting equipped with a packing gland and sealing nut. The capillary was inserted at right angles to the flow so that the upper end of the tube was located on the axis of the pipe. The lower end was inserted in the stopcock 3, which was connected to the calibrated sealed vessel 4 by a flow passage considerably greater than the internal diameter

\*The rheology of colloidal systems has been developed in detail.

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